OPTIMAL ESTIMATION OF THE ROUGHNESS COEFFICIENT USING UNSTEADY OPEN CHANNEL FLOW DATA

A Thesis Submitted in Partial
Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

by

A.NARAYANA

to

Department of Civil Engineering
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

JULY, 1996

Dedicated

to

APPA

AND

AMMA

CERTIFICATE

It is certified that the work contained in the thesis entitled *Optimal estimation of the roughness coefficient using unsteady open channel flow data* by *A.Narayana*, has been carried out under our supervision and that this work has not been submitted elsewhere for a degree.

Dr. Bithin Datta
Professor
Department of Civil Engineering
Indian Institute of Technology,
Kanpur

Dr. B.S. Murty
Assistant Professor
Department of Civil Engineering
Indian Institute of Technology,
Kanpur

JULY 1996

ABSTRACT

The inverse problem of estimating the open channel flow roughness (Mannings n) is solved using an optimization algorithm. Measurements of flow variables (depth of flow and discharge) at different locations and times are used as inputs to the optimization model. Unsteady flow conditions are considered. Both, single value of Mannings "n" throughout the channel length and multiple values of "n" at different channel reaches are considered. The Sequential Quadratic Programming algorithm is used to solve the optimization model. The optimization model embeds the finite difference forms of the governing equations of open channel flow. The Preissmann scheme is used for discretization of the flow equations.

The performance of the developed optimization model is evaluated for different scenarios of data availability. Solution results of the model for illustrative problems establish the potential applicability of the proposed approach.

ACKNOWLEDGEMENT

It gives me immense pleasure to express my feeling of gratitude and admiration to my knowledgeable guides Dr. Bithin Datta and Dr. B.S. Murty for their constant support and confidence instilled in myself, which enabled me to complete this work and go through this challenge. Their deep discerning perceptibility helped me to understand the finer points in this field and I once again express my sincere appreciation for the same.

I would like to acknowledge all the faculty members in Hydraulics and Water Resources specialization of Civil Engineering Department for their contribution of preparing a balanced, resourceful course work, standing me in good stead in the course of my work. Also I would like to thank all my batch-mates and my senior collegues for making my stay here a memorable one.

Finally, I thank my parents for their blessings and encouragement received at every juncture, which has helped me to complete my work smoothly.

A.NARAYANA

Contents

List of Figures

List of Tables

Chapter 1

INTRODUCTION

1.1 Introduction

An accurate estimation of the roughness coefficient is of prime importance to any study involving open-channel flow such as:

- Flood estimation, routing and damage mitigation.
- Optimal design and operation of canal and surface irrigation systems.
- Estimation of degradation and aggradation of channels due to natural and human-made causes and
- Other river engineering studies.

Mathematical models which are routinely used in these studies require calibration with regards to the roughness coefficient before they are applied. In fact, the whole process of effective utilization of surface water resources requires a knowledge of the roughness coefficient.

Estimation of the roughness coefficient or the Mannings "n" for a natural channel is not a trivial task because it depends on various factors such as:

1. surface roughness.
2. vegetation.
3. channel irregularity.
4. obstructions.
5. channel alignment.
6. sedimentation and scouring.

Many empirical procedures (French, 1990) have been suggested in the past for estimating the value of n. The Soil Conservation Service (SCS) method (Urquhart 1975) for estimating "n" involves the selection of a basic "n" value for a uniform, straight and regular channel in a native material and then modifying this value by adding correction factors. These correction factors are determined by a critical consideration of some of the factors mentioned above. The selection of correction factors in this method is highly heuristic.

A second method of estimating "n" for a channel involves the use of tables of values. Chow (1959) presented an extensive table of "n" values for various types of channels. In this table, a minimum, a normal and a maximum value of "n" are stated for each type of channel. This method gives satisfactory results for human-made channels. However, it is not suitable for estimating the roughness coefficient of the natural channels.

It is the belief of the U.S. Geological Survey that photographs of channels of known resistance together with a summary of the geometric and hydraulic parameters which define the channel for a specified flow rate, can be useful in estimating the resistance coefficient (Barnes 1967). The U.S. Geological Survey maintains a program which trains engineers in the estimation of channel resistance coefficient. It is claimed that the trained engineers can estimate resistance coefficeints with an accuracy of $\pm 15\%$ under most conditions. However, it should be noted that this is not a formal procedure and a satisfactory application is highly dependent on the person's previous experience.

From a theoritical viewpoint, a value of the resistance coefficient can be estimated from the velocity and the depth measurements because these depend on the roughness. Chow (1959) suggested the following equation for estimating "n" from the velocity measurements at a cross-section.

$$n = \frac{(x-1)h^{\frac{1}{6}}}{6.78(x+0.95)} \tag{1.1}$$

where:

$$x = \frac{U_{0.2}}{U_{0.8}} \tag{1.2}$$

$U_{0.2}$=Velocity at $\frac{2}{10^{th}}$ of the depth
$U_{0.8}$=Velocity at $\frac{8}{10^{th}}$ of the depth
h=depth of flow

The above equations (1.1 and 1.2) are applicable only when channel is wide and the velocity measurements are made for uniform flow conditions. However, the idea of estimating the Mannings "n" from the field measurements of discharge and depth i.e. the "inverse problem" can be used for non-uniform flow conditions in irregular channels. The Standard Step Method (Chow 1959, Subramanya 1984) can be used to simulate the steady, non-uniform open channel flow. The Mannings "n" for the channel may be determined by a trial and error procedure such that the difference between the simulated and the measured water surface profile is minimum. Two points should be noted here with regards to the inverse problem. Firstly, a formal search technique (Rao, 1984) may be used for determining the Mannings "n" instead of a trial and error procedure. This lends formality to the calibration procedure, decreases the arbitrariness and gives measure of likely error in the estimated value. Secondly, the best solution is expected when an abundance of data exists. Fortunately, the unsteady flow situation can yield an abundance of data, since simple stage measurements over a period of time can give many data points.

In the present study, a formal approach is developed for estimating the Mannings "n" from the measured transient flow data. The developed methodology is based

on a nonlinear optimization technique in which the flow equations are embedded as equality constraints.

1.2 Literature review

The literature relating to identification and estimation of parameters for unsteady open channel flow are sparse. The determination of parameters in a set of partial differential equations subjected to specified boundary conditions and some measurements is generally known as the "Inverse Problem". Significant amount of work relating to parameter identification in the groundwater has been reported. Solution techniques for aquifer parameter identification problem have included,

1. Quasilinearation used by Yeh and Tauxe (1971),
2. Linear programming used by Kleinecke (1971),
3. Multiple-objective linear programming used by Neumann (1973),
4. Marquaardt's nonlinear estimation algorithm used by Garay, Haines and Das (1976).

Becker and Yeh (1972) proposed a methodology for identification of parameters in unsteady open channel flow using the influence coefficient approach. The parameters chosen for identification was the channel roughness coefficient and exponent of the hydraulic radius in the empirical friction slope relation. The influence coefficients define the effect of unit changes in the hydraulic radius exponent and the channel roughness coefficient on the errors between observed and simulated values of flow variables like velocity and depth. The methodology consists of solving for the objective of minimizing the sum of squares of errors between observations and simulated numerical solution of the governing equations based on some estimated values of parameters. The algorithm starts by specifying initial estimates for the parameters in the feasible region. The governing equation are solved by the explicit (staggered) finite-difference scheme and

the function defining the error is evaluated. The parameters are modified based on the evaluated objective function and the influence coefficient matrix. This procedure is repeated with new influence coefficients computed for every iteration. This procedure may perform well for lesser number of parameters but proves to be a tedious model for estimating a large number of parameters, as simulation has to be carried out several number of times. Also the stability and convergence criteria have to be satisfied as an explicit scheme is used while solving the governing equations.

Becker and Yeh (1973a) have used unsteady state flow conditions for identification of multiple reach channel parameters by using influence coefficient approach. The parameters estimated were the bed slope and the Mannings "n" at every reach. Steady state flow conditions are assumed to be attained prior to the initiation of an input flood hydrograph. Steady state computation assumed uniform flow in each reach, modified by backwater conditions in the neighbourhood of and some distance upstream of each junction. The relatively large rate of change of flow conditions near the junction necessitates a finer spacial interval in that neighbourhood than elsewhere, in order to provide sufficient accuracy and to avoid numerical oscillations that may otherwise arise. Once initial conditions are determined, unsteady flow equations are solved using staggered explicit finite-difference discretization for interior grid points and for the boundary points. The friction parameters for each reach of channel are identified using the influence coefficient approach in a sequential manner. Hence, it becomes a tedious procedure.

The applicability of linear programming together with influence coefficient approach for unsteady open channel flow parameter identification is presented in Becker and Yeh (1973b). The Simplex linear programming algorithm is used in this model. The Minmax criteron (Minimizing the maximum of absolute value of the errors) is combined with influence coefficient technique to yield a linear programming formulation suitable for identification of the parameters of unsteady open channel flow. The parameters estimated are the Mannings "n", exponent of hydraulic radius and lateral inflow.

Fread and Smith (1978) proposed a methodology for estimating the parameter "n" that varies with space and with stage or discharge and applied the methodology to an hypothetical ideal channel, a mainstem river and a dendritic river system. This method uses a gradient type modified Newton-Raphson algorithm to improve the initial trial function, such that difference between observed and computed stages is minimized. The computed stages are obtained from a four-point implicit finite difference solution of one dimensional unsteady flow equations subject to upstream and downstream boundary conditions of observed discharge and stage hydrograph respectively.

Wasantha Lal (1995) used the Singular value decomposition method to calibrate the Mannings roughness coefficient in a one-dimensional unsteady flow model. The method is used to solve for the parameters after formulating the calibration problem as a generalized linear inverse problem. Calibration was repeated with different numbers of parameter groups to determine the variation of the output error and uncertainity of the parameters with the parameter dimension. Like the earlier methods of parameter identification, this method also uses the influence coefficient approach to improve the assumed values of the parameters which are being estimated. However, it employs the Singular value technique so that underdetermined, overdetermined, even or mixed determined problems could be solved. Using this approach, optimal grouping of sensitive parameters can be determined.

In all these previously mentioned works, the parameters are estimated by solving the governing equations in a separate solver for estimated values of the parameters, outside the optimization model or algorithm. The aim of the present study is to estimate the parameter "n" by directly embedding the finite-difference discretization of the system of governing equations for the open channel flow into the nonlinear optimization model. As the unsteady flow equations are a set of nonlinear partial differential equations, using a nonlinear optimization technique for solution is justified. In this study, the flow equations are discretized using an implicit finite-difference scheme (Preissmann scheme). The advantage of using this scheme is that the stability and convergence criteria need not be checked. The nonlinear optimization technique used in this study

is the Sequential Quadratic Programming (SQP), which is available in Numerical Algorithm Group Library (NAG, 1990). The main advantage of using an embedded optimization model for solving the inverse problem is that an iterative procedure is not necessary. The flow hydraulics is directly simulated within the optimization model by using the finite-difference equations as equality constraints. In the iterative method, when the simulation is outside the optimization model, convergence to an optimum solution may prove to be difficult if a large number of parameters are to be identified. Also the embedding approach results in an elegent formulation and has the potential of incorporating various complex flow systems and boundary conditions. However, the computational limitations may hinder its universal application.

In this study, the flow equations are discretized using an Implicit finite-difference scheme (Preissmann scheme). The advantage of using this schemeis that the stability and convergence criteria need not be checked.

1.3 Objective of the study

Objectives of the present study are:

- To develop an embedded nonlinear optimization model for estimating the roughness coefficient (Mannings "n") of an open channel using transient flow observation data.
- To test the developed model for solvability using simulated data as observed data for a hypothetical channel.
- To evaluate the performance of the developed model through illustrative examples.

1.4 Organization of the thesis

The thesis is divided into four chapters:

1. Chapter I contains the introduction to this study and a discussion of the relevent literature.
2. Chapter II presents the methodology and also discusses the optimization algorithm used.
3. Solution results and performance evaluations are dicussed in Chapter III.
4. Conclusions are presented in Chapter IV.

Chapter 2

METHODOLOGY

2.1 Introduction

The proposed model for estimating the Mannings roughness coefficient "n" involves the use of a nonlinear optimization model with the governing equations of the flow forming the equality constraints. Unsteady shallow water flow equations representing the mass and momentum conservation are the governing equations. These are a set of nonlinear hyperbolic partial differential equations which are discretized using the implicit Preissmann finite-difference scheme (Chaudhry 1993). This scheme is used in this study because large computational time steps can be used in the discretization. The optimization algorithm used in this work is the Sequential Quadratic Programming (SQP) which is available in the NAG Library (NAG, 1990) as subroutine E04UCF. It is designed to minimize an arbitrary smooth nonlinear objective function subjected to constraints, which may include simple bounds on the variables, linear constraints and smooth nonlinear constraints separately. Since the objective functions and constraint functions are smooth i.e., at least twice differentiable, SQP can be used.

2.2 Governing Equations

One-dimensional, unsteady shallow water flow equations for a prismatic, wide rectangular channel are used as the governing equations in this study. It is however possible to extend the methodology to general one-dimensional flow in irregular channels. The governing equations adopted are based on the following assumtions:

- The pressure distribution is hydrostatic.
- The channel bottom slope is small, so that the flow depths measured normal to the channel bottom and measured vertically are approximately the same.
- The flow velocity over the entire channel cross-section is uniform.
- The channel is prismatic.
- The head losses in unsteady flow may be simulated by using the steady-state resistance laws, such as the Mannings equation.

The governing equations represent the mass and momentum conservation and are written as: (Subramanya 1984; Chaudhry 1993)

Continuity equation

$$\frac{\partial y}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{2.1}$$

Momentum equation

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}(\frac{q^2}{y} + \frac{gy^2}{2}) = gy(s_o - s_f) \tag{2.2}$$

where:
$y(x,t)$=flow depth,
$q(x,t)$=discharge per unit width of channel,
s_o=slope of channel,
s_f=friction slope,
x=distance along the flow direction

t=time.

The friction slope, s_f is given by the Mannings equation.

$$s_f = \frac{q^2 n^2}{y^{\frac{10}{3}}} \tag{2.3}$$

in which, n = Mannings roughness coefficient.

In the discretized form, the partial derivatives and other coefficients are approximated using the Preissmann scheme (Chaudhry, 1993).

2.2.1 Preissmann Scheme for Discretization

Equations (2.1) and (2.2) have to be discretized using a finite-difference grid (Fig. 2.1) before they are used in either a simulation model or in an optimization model as equality constraints.

In the following discussions, subscript i refers to the grid point in the x-direction and the superscript k refers to the time index. The partial derivatives and other coefficients are approximated as follows, using the Preissmann scheme (Chaudhry, 1993).

$$\frac{\partial f}{\partial t} = \frac{(f_i^{k+1} + f_{i+1}^{k+1}) - (f_i^k + f_{i+1}^k)}{2\Delta t} \tag{2.4}$$

$$\frac{\partial f}{\partial x} = \frac{\alpha(f_{i+1}^{k+1} - f_i^{k+1})}{\Delta x} + \frac{(1-\alpha)(f_{i+1}^k - f_i^k)}{\Delta x} \tag{2.5}$$

$$f = \frac{1}{2}[\alpha(f_{i+1}^{k+1} + f_i^{k+1}) + (1-\alpha)(f_{i+1}^k + f_i^k)] \tag{2.6}$$

in which,

Δx=grid spacing,

Δt=computational time step and

α=weighting coefficient.

In the partial derivatives (Equations 2.4, 2.5 and 2.6), f refers to both q and y; f as a coefficient stands for s_f and q.

The scheme is stable if $0.5 \leq \alpha \leq 1.0$. A value of 0.8 is used in the present study. Substitution of the preceding finite difference approximations into Eqs.(2.1) and (2.2)

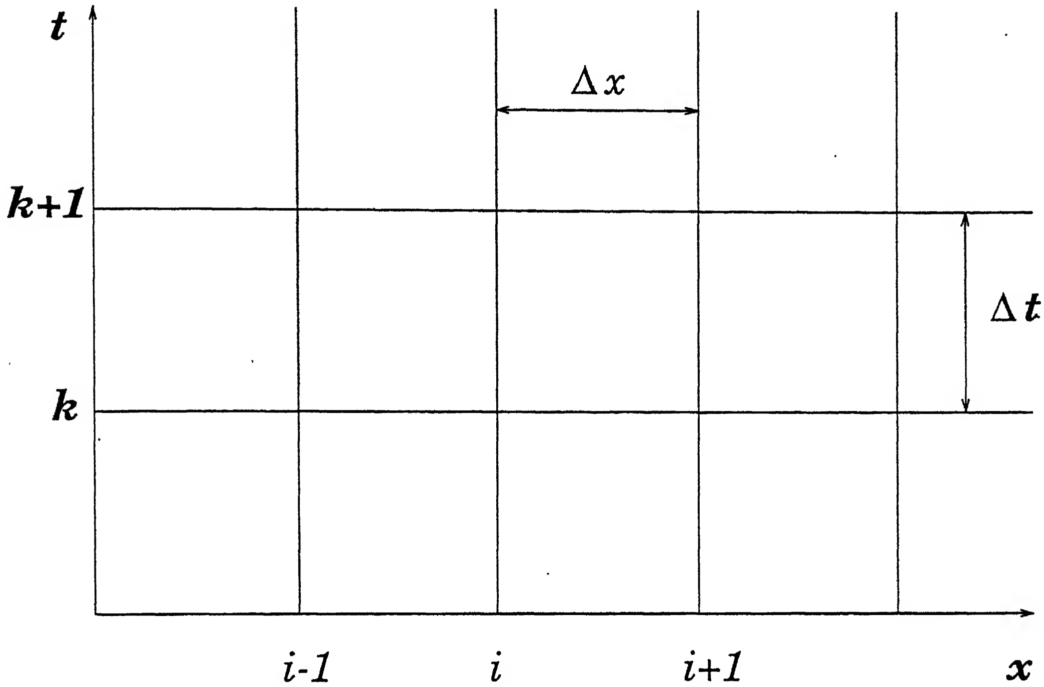

Figure 2.1: **Finite-difference grids**

and subsequent rearrangement leads to the following algebraic equations valid at any node i, with "N" being the total number of nodes being considered.

<u>**Continuity Equation**</u>

$$\frac{(y_{i+1}^{k+1}+y_i^{k+1})-(y_{i+1}^{k}+y_i^{k})}{2\Delta t}+\frac{\alpha(q_{i+1}^{k+1}y_{i+1}^{k+1}-q_i^{k+1}y_i^{k+1})}{\Delta x}+\frac{(1-\alpha)(q_{i+1}^{k}y_{i+1}^{k}-q_i^{k}y_i^{k})}{\Delta x}=0 \tag{2.7}$$

for i = 1 to N - 1

<u>**Momentum Equation**</u>

$$\frac{(q_{i+1}^{k+1}+q_i^{k+1})-(q_{i+1}^{k}+q_i^{k})}{2\Delta t}+\frac{\alpha}{\Delta x}[\frac{(q_{i+1}^{k+1})^2}{h_{k+1}^{k+1}}-\frac{(q_i^{k+1})^2}{h_i^{k+1}}]+\frac{(1-\alpha)}{\Delta x}[\frac{(q_{i+1}^{k})^2}{h_{i+1}^{k}}-\frac{(q_i^{k})^2}{h_i^{k}}]+$$

$$\frac{g\alpha}{2\Delta x}[(h_{i+1}^{k+1})^2-(h_i^{k+1})^2]+\frac{g(1-\alpha)}{2\Delta x}[(h_{i+1}^{k})^2-(h_i^{k})^2]+\frac{g\alpha n^2}{2}[\frac{(h_{i+1}^{k+1}(q_{i+1}^{k+1})^2}{(h_{i+1}^{k+1})^{\frac{10}{3}}}+$$

$$\frac{h_i^{k+1}(q_i^{k+1})^2}{(h_i^{k+1})^{\frac{10}{3}}}] - gs_o[\frac{\alpha}{2}(h_{i+1}^{k+1} + h_i^{k+1}) + \frac{(1-\alpha)}{2}(h_{i+1}^k + h_i^k)] + \frac{g(1-\alpha)n^2}{2}$$

$$[\frac{h_{i+1}^k(q_{i+1}^k)^2}{(h_{i+1}^k)^{\frac{10}{3}}} + \frac{h_i^k(q_i^k)^2}{(h_i^k)^{\frac{10}{3}}}] = 0 \quad (2.8)$$

for i = 1 to N - 1

2.3 Formulation of the Problem

The Inverse problem is formulated as nonlinear optimization model, with the resistance coefficient (n) as an unknown variable. The model consists of the open channel flow governing equations, boundary conditions and observed values of flow variables (depth and discharge). The governing equations are the continuity and momentum equations. These governing equations are included as equality constraints in the non-linear optimization algorithm, in the discretized form as shown in eqn.(2.7) and eqn.(2.8). The continuity equation at each node forms the linear equality constraints and momentum equation forms the non-linear equality constraints. The upstream boundary conditions are the specified discharge hydrograph at the upstream end. The downstream boundary conditions follow the steady-state resistance laws. Hence the upstream boundary conditions form the linear constraints and downstream boundary conditions form the nonlinear constraints. The objective function is sum of squares of the differences between the estimated and measured values of the flow variables. The formulation of the Inverse problem can be summarized as shown below:

Objective Function: The objective function minimizes the sum of squares of the differences between the observed flow variable values and the model simulated flow variable values.

Min

$$\sum_1^N [\frac{x(O) - x(M)}{x(M)}]^2 \quad (2.9)$$

Where:

x(O) are the simulated depth or discharge values

x(M) are the measured depth or discharge values

Linear Constraints: The finite difference forms of the continuity equation and the upstream boundary conditions constitute the linear equality constraints of the model. These constraints are:

$$\frac{(y_{i+1}^{k+1}+y_i^{k+1})-(y_{i+1}^{k}+y_i^{k})}{2\Delta t}+\frac{\alpha(q_{i+1}^{k+1}y_{i+1}^{k+1}-q_i^{k+1}y_i^{k+1}}{\Delta x}+\frac{(1-\alpha)(q_{i+1}^{k}y_{i+1}^{k}-q_i^{k}y_i^{k})}{\Delta x}=0 \tag{2.10}$$

$$for\ \ i=1\ to\ N{-}1; k=1\ to\ p\ \ (p=Total\ number\ of\ time\ steps)$$

$$q_1^{k+1}=Specified\ by\ inflow\ hydrograph;\quad for\ k=1\ to\ p \tag{2.11}$$

Non-Linear Constraints: The finite difference form of the momentum equation and the downstream boundary conditions constitute the nonlinear equality constraints of the model. These constraints are:

$$\frac{(q_{i+1}^{k+1}+q_i^{k+1})-(q_{i+1}^{k}+q_i^{k})}{2\Delta t}+\frac{\alpha}{\Delta x}[\frac{(q_{i+1}^{k+1})^2}{h_{k+1}^{k+1}}-\frac{(q_i^{k+1})^2}{h_i^{k+1}}]+\frac{(1-\alpha)}{\Delta x}[\frac{(q_{i+1}^{k})^2}{h_{i+1}^{k}}-\frac{(q_i^{k})^2}{h_i^{k}}]+$$

$$\frac{g\alpha}{2\Delta x}[(h_{i+1}^{k+1})^2-(h_i^{k+1})^2]+\frac{g(1-\alpha)}{2\Delta x}[(h_{i+1}^{k})^2-(h_i^{k})^2]+\frac{g\alpha n^2}{2}[\frac{h_{i+1}^{k+1}(q_{i+1}^{k+1})^2}{(h_{i+1}^{k+1})^{\frac{10}{3}}}+$$

$$\frac{h_i^{k+1}(q_i^{k+1})^2}{(h_i^{k+1})^{\frac{10}{3}}}]-gs_o[\frac{\alpha}{2}(h_{i+1}^{k+1}+h_i^{k+1})+\frac{(1-\alpha)}{2}(h_{i+1}^{k}+h_i^{k})]+\frac{g(1-\alpha)n^2}{2}$$

$$[\frac{h_{i+1}^{k}(q_{i+1}^{k})^2}{(h_{i+1}^{k})^{\frac{10}{3}}}+\frac{h_i^{k}(q_i^{k})^2}{(h_i^{k})^{\frac{10}{3}}}]=0 for i=1 to N-1; k=1 to p \tag{2.12}$$

$$q_N^{k+1}=\frac{(y_N^{k+1})^{\frac{5}{3}}s_o^{\frac{1}{2}}}{n}\quad for\ k=1\ to\ p \tag{2.13}$$

In addition, the following physical constraint is specified:

$$n>0 \tag{2.14}$$

The above optimization model is nonlinear because of the nonlinear objective function and nonlinear constraints.

2.4 Optimization Algorithm

The above optimization model is nonlinear because of the nonlinear objective function and nonlinear constraints. The nonlinear optimization model is solved using the Sequential Quadratic Programming (SQP) algorithm, as coded in the NAG library (NAG, 1990). The salient features of this algorithm as described in the NAG Users Manual is presented here.
The SQP algorithm is used to solve the general optimization problem:

$$\textbf{Min} : F(x) \tag{2.15}$$

$$\textbf{Subject to} : l \leq \left\{ \begin{array}{c} x \\ A_L x \\ C(x) \end{array} \right\} \leq u \tag{2.16}$$

where F(x) is the nonlinear objective function, A_L is an n_L x n coefficient matrix, C(x) is an n_N element vector of nonlinear constraint. The objective function and constraint functions are assumed to be smooth i.e., atleast twice-continuously differentiable.

At the solution of eq(2.16), some of the constraints will be active, i.e., satisfied exactly. An active simple bound constraint implies that the corresponding variable is fixed at its bound, and hence the variables are partitioned into fixed and free variables. A point x is a *first-order Kuhn-Tucker point* for eq(2.16) if the following conditions hold (Powell, 1974):

1. x is feasible;

2. there exists vectors ξ and λ (the Lagrange multiplier vectors for the bound and general constraints) such that

$$g = C^T \lambda + \xi \tag{2.17}$$

where g is the gradient of F evaluated at x, and $\xi_j = 0$ if the jth variable is free.

3. The Lagrange multiplier corresponding to an inequality constraint active at its lower bound must be non-negative, and non-positive for an inequality constraint active at its upper bound.

There are several ways of organizing the matrix calculations of active set algorithms for quadratic programming. By working with the upper triangular matrix method for quadratic programming, it is possible to carry out the matrix operations economically. The advantage of this approach (Powell, 1983) is that there is a substantial saving of computer storage, because the orthogonal matrix is not required. All these features are included in E04UCF subroutine (NAG, 1990), which is used in this study. The basic structure of E04UCF involves major and minor iterations. The major iterations generate a sequence of iterates x_k that converge to x^*, a first-order Kuhn-Tucker Point of eq(2.16). At a typical major iterations, the new iterate $\bar{x}$ is defined by

$$\bar{x} = x + \alpha p \tag{2.18}$$

where x is the current iterate, $'\alpha'$ the steplength and p is the search direction. Also associated with each major iteration are estimates of the Lagrange multipliers and a prediction of the active set.
The search direction p is the solution of a quadratic programming subproblem of the form,

$$\text{Minimize: } g^T p + \tfrac{1}{2} p^T H_p$$

$$Subject\ to: \bar{l} \leq \left\{ \begin{array}{c} p \\ A_L p \\ A_N p \end{array} \right\} \leq \bar{u} \tag{2.19}$$

where g is the gradient of function at x, the matrix H is a positive definite quasi-Newton approximation to the Hessian of the Lagrangian function. A_N is the jacobian matrix of C evaluated at x. The matrix in eq(2.18) is a positive-definite quasi-Newton approximation to the Hessian of the Lagrangian function. (Dennis and Schnabel, 1981). At the end of each of major iteration, a new Hessian approximation $\bar{H}$ is defined as a

rank-two modification of H.

$$\bar{H} = H - \frac{1}{s^T H s} hss^T H + \frac{1}{y^T s} yy^T \tag{2.20}$$

where $s = \bar{x} - x$ (the change in x)
In E04UCF, H is required to be positive-definite. If H is positive-definite, $\bar{H}$ defined by eq(2.13) will be positive-definite if and only if $y^T s$ is positive. Ideally, y in eq(2.20) would be taken as y_L, the change in gradient of the Lagrangian function

$$y_L = \bar{g} - \bar{A}_N^T \mu_N - A_N^T \mu_N \tag{2.21}$$

Where μ_N denotes the QP multipliers associated with the nonlinear constraints of the original problem. If $y_L^T s$ is not sufficiently positive, an attempt is made to perform the update with vector y of the form

$$y = y_L + \sum_{i=1}^{m_N} \omega_i (a_i(\bar{x}) c_i(\bar{x}) - a_i(x) c_i(x)) \tag{2.22}$$

where $\omega_i \geq 0$. If no succh vector can be found, the update is performed with a scaled y_L. After computing the search direction, each major iteration proceeds by determining a steplength α in eq(2.18) that produces a 'sufficient decrease' in the augmented Lagrangian merit function

$$L(x, \lambda, s) = F(x) - \sum_i \lambda_i (c_i(x) - s_i) + \frac{1}{2} \sum_i \rho_i (c_i(x) - s_i)^2, \tag{2.23}$$

where x, λ, and s vary during the linesearch. The summation terms in the above equation involve only the nonlinear constraints. The vector λ is an estimate of the Lagrange multipliers for the nonlinear constraints of eq(2.9). Therefore each iteration includes,

1. The solution of a quadratic programming subproblem.
2. A line search with an augmented Lagrangian merit function and
3. A quasi-Newton update of the approximate Hessian of Lagrangian function

Chapter 3

PERFORMANCE EVALUATION OF THE PROPOSED MODEL

3.1 Simulation of Flow Measurement Data

Performance of the developed optimization model for the estimation of the roughness coefficient was evaluated using illustrative example problems for a hypothetical open channel. In this study, the parameter "n" is estimated for a single reach channel and multiple reach channels. As the main aim is to verify the capability of the model to estimate the roughness coefficient, without going into the broader issues of measurement errors and modelling uncertainities, which are not incorporated. In the illustrative examples, identical initial and boundary conditions are implemented in the simulation and optimization models. Flow and discharge measurement data in space and time were simulated using a numerical simulation model for unsteady open channel flow. The Preissmann discretization scheme was utilized and the solution scheme as given by Chaudhry (1993) was adopted for simulation. Using the simulated observation data, simulated with known values of "n" also ensures that the estimated "n" values and the actual "n" can be correctly compared.

3.2 Illustrative Problem

A rectangular channel with a constant bed slope of 0.0004 is considered. Initial steady-state for a single reach channel are given as: y = 2.56m and q = $4m^3$/sec/m width of the channel. The reach length is 12km and observation stations are assumed to be located at 2km interval from the upstream boundary. An incoming triangular hydrograph is assumed at the upstream end and observations of discharges and depths are simulated by direct numerical solution of eqs(2.1) and (2.2) using the Preissmann Implicit finite-difference scheme. The simulation is carried out for three time steps and the results are tabulated at each grid locations. There are six nodes for each of the time steps as shown in Fig.(3.1). The grid locations are indicated in the Fig.(3.1). The succeeding sections discuss the performance evaluation results.

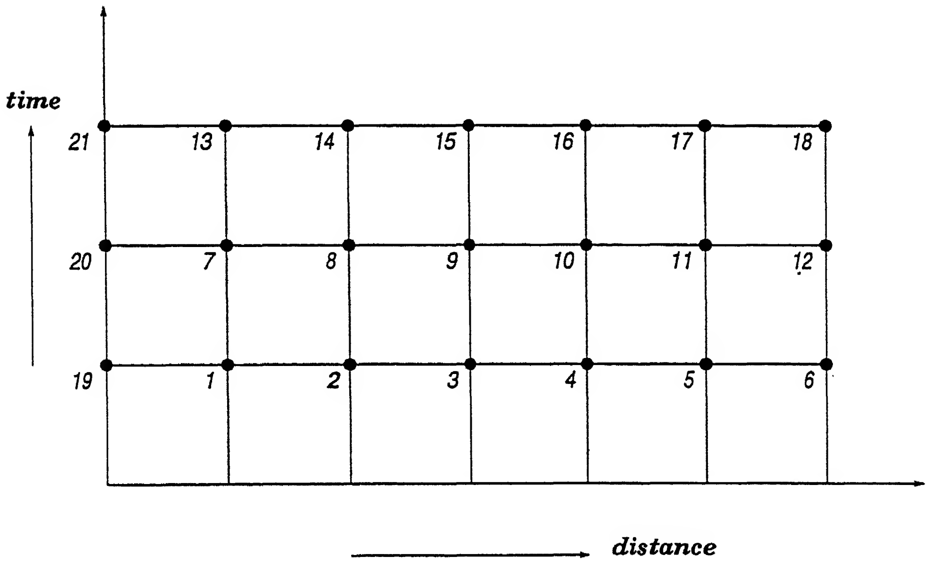

Figure 3.1: Grid identification scheme

3.3 Verification of the Model for a Single Reach Channel

The ability of the developed model to correctly estimate Mannings "n" for a single reach channel is first established. The flow observation data for unsteady flow in a channel with a single representative value of "n" is simulated first. The value of "n" is assumed to be 0.023 in the simulation model. These simulated observations are used as inputs to the optimization model.
The discretized form of flow equations [eq(2.7) and eq(2.8)] for each box formed by four nodes (Fig. 3.1), results in eighteen linear and eighteen non-linear constraints respectively. The boundary conditions as described below are also needed to be specified. The Model solves these flow equations, for given boundary conditions and the flow measurments data (here the simulated data), to estimate the parameter "n" (roughness coefficient).

Initial Conditions:

The initial conditions are y(i) = 2.56m and q(i) = 4.0m^2/sec for all i = 1 to 7. These initial conditions are directly incorporated into the discretized equations (eqs. 2.7 and 2.8) for the time level k = 1.

Upstream Boundary Conditions:

The discharge hydrograph shown in Fig. 3.2 forms the upstream boundary conditions. This translates into following linear equality constraints.

$$x(19,2) = 4.6667m^2/sec. \tag{3.1}$$

$$x(20,2) = 5.3333m^2/sec. \tag{3.2}$$

$$x(21,2) = 6.0000m^2/sec. \tag{3.3}$$

Downstream Boundary Conditions:

The downstream boundary conditions are the steady-state resistance laws.

$$x(6,2) = \frac{x(6,1)^{\frac{5}{3}} so^{\frac{1}{2}}}{n} \tag{3.4}$$

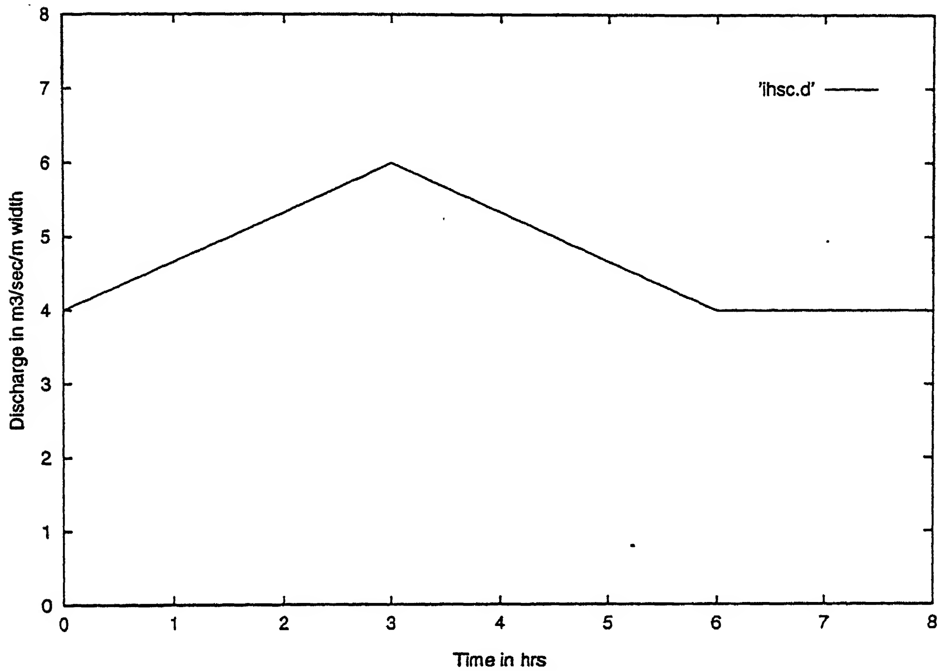

Figure 3.2: **Incoming flow hydrograph at the upstream end of the channel (for single reach channel)**

$$x(12,2) = \frac{x(12,1)^{\frac{5}{3}} so^{\frac{1}{2}}}{n} \tag{3.5}$$

$$x(18,2) = \frac{x(18,1)^{\frac{5}{3}} so^{\frac{1}{2}}}{n} \tag{3.6}$$

Where:

x(19,2), x(20,2), x(21,2) = Represents the discharge at grids 19, 20 and 21.

x(6,2), x(12,2), x(18,2) = Represents the discharge at grids 6,12 and 18

x(6,1), x(12,1), x(18,1) = Represents the depth at grids 6, 12 and 18

so = Bottom slope of the channel

n = Mannings roughness coefficient

The Objective function is the sum of squares of the difference between the model simulated values of the flow variables for the optimal estimate of "n" and the measured data of depth and discharge at different nodes. The depth and discharge are treated as variables along with the parameters to be estimated i.e., the Mannings "n". Some initial estimate of all these variables are specified to start the optimization algorithm.

Embedding technique may prove to be an elegent method for Inverse problems, especially when there is abundance of data. Fortunately unsteady flow conditions yield numerous data points. To study the robustness of the model, different sets of measured data are used. Table.(3.1) presents the simulated measurement data for a single reach channel. Actual value of Mannings "n" used in the measurement simulation model is 0.023. The six scenarios chosen are presented in Table.(3.2). From the results of the estimated parameter, it is clear that as number of data points are less, the estimated parameter is questionable. Also the optimum value of discharge and depth are almost identical to that of the simulated measurement data, when large number of data points are input in the optimization model. [Refer to fig.(3.3) and fig.(3.4)]. For scenarios 1 to 3, the estimated and actual values of "n" are identical. However, when much smaller number of measurement data points are used, scenarios 4, 5 and 6, the maximum error in the estimation of "n" is about 15 percent only. These results establish the correctness of the model solution at least for the simple and ideal case of a single reach channel with no measurement errors.

Table 3.1: **Simulated observation Data for a Single Reach Channel**

Grid Point	Depth in metres	Discharge m^3/sec/m width	Grid Point	Depth in meters	Discharge m^3/sec/m width
1	2.66	4.54	2	2.63	4.43
3	2.61	4.35	4	2.59	4.28
5	2.57	4.23	6	2.56	4.18
7	2.89	5.21	8	2.85	5.09
9	2.81	4.96	10	2.77	4.85
11	2.74	4.74	12	2.73	4.63
13	3.11	5.88	14	3.07	5.76
15	3.02	5.64	16	2.99	5.51
17	2.96	5.39	18	2.95	5.26
19	2.70	4.67	20	2.93	5.33
21	3.14	6.00			

Table 3.2: Scenarios Studied for Single Reach Channel

Scenario No.	No. of Data Points for Depth	No. of Data Points for Discharge	Estimated value of "n"	Actual Value of "n"
1	21	21	0.023	0.023
2	17	17	0.023	0.023
3	12	12	0.023	0.023
4	5	5	0.02313	0.023
5	21	0	0.0262	0.023
6	0	21	0.0232	0.023

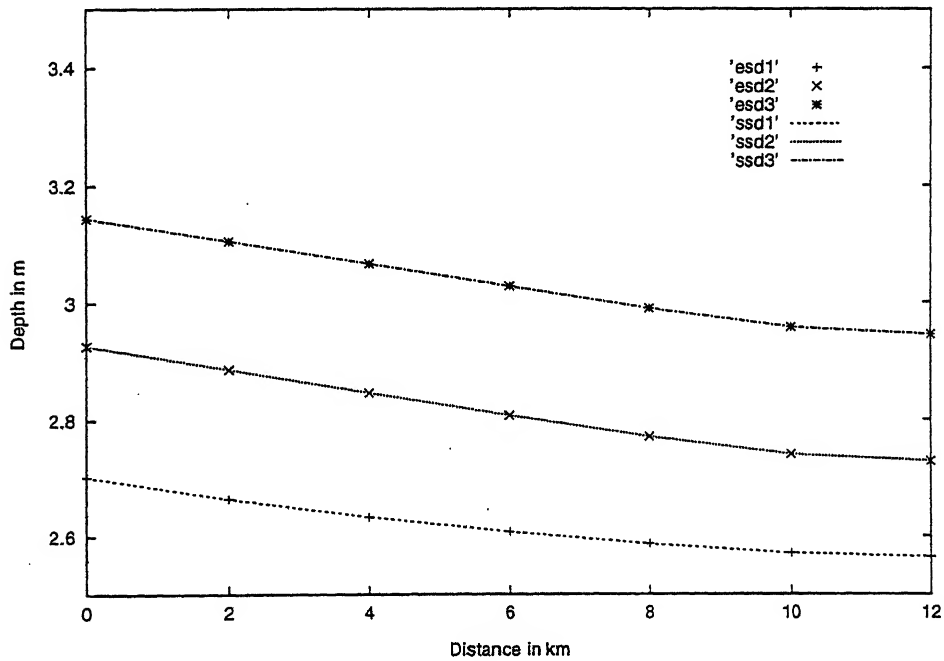

Figure 3.3: Comparison of flow depths as observed (Simulated) and obtained as solution to the optimization model (Single reach, Scenario:1)

3.4 Estimation of Mannings "n" for a Multiple Reach Channel

The formulation is similar to the earlier case of a single reach. The difference is that the Mannings "n" to be estimated at each reach is incorporated as a variable

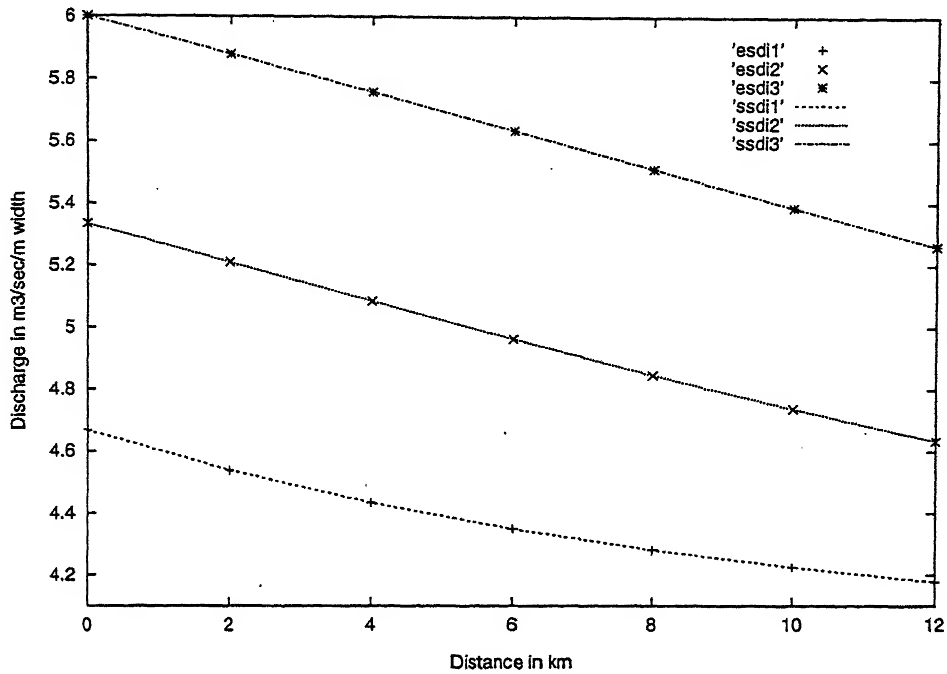

Figure 3.4: **Comparison of discharge as observed (Simulated) and obtained as solution to the optimization model (Single reach, Scenario:1)**

in the optimization model. The model is tested for different sets of simulated flow measurement data (depth and discharge). A rectangular channel with six reaches, each having the same slope of 0.0004 and with different Mannings "n" is chosen as an illustrative example for estimation of "n" values for multiple reaches. The Mannings "n" used for simulation at the six reaches of the channel are 0.024, 0.025, 0.021, 0.018, 0.019 and 0.023 respectively. The initial conditions used in this case are y(i) = 5.56m and q(i) = 20.0m^2/sec. The discharge hydrograph is shown in fig.(3.5). Table.(3.4) presents the simulated data for a Multiple reach Channel. The model is again tested for different measurement data and scenarios as shown in tables 3.5 and 3.6 respectively.

Table 3.3: **Notation used in fig.(3.3) and fig.(3.4)**

Notation used	Represents for a Single Reach Channel
esd1	Depth estimated for 1st time step
esd2	Depth estimated for 2nd time step
esd3	Depth estimated for 3rd time step
ssd1	Simulated measured depth for 1st time step
ssd2	Simulated measured depth for 2nd time step
ssd3	Simulated measured depth for 3rd time step
esdi1	Discharge estimated for 1st time step
esdi2	Discharge estimated for 2st time step
esdi3	Discharge estimated for 3st time step
ssdi1	Simulated measured discharge for 1st time step
ssdi2	Simulated measured discharge for 2nd time step
ssdi3	Simulated measured discharge for 3rd time step

Table 3.4: **Simulated observation data for a Multiple reach Channel**

Grid Point	Depth in metres	Discharge m^3/sec/m width	Grid Point	Depth in meters	Discharge m^3/sec/m width
1	6.40	20.94	2	5.96	20.52
3	5.92	20.25	4	6.09	19.81
5	5.92	19.33	6	6.33	18.81
7	6.74	23.32	8	6.45	23.14
9	6.45	22.85	10	6.59	22.61
11	6.57	22.33	12	6.95	22.01
13	7.04	24.79	14	6.75	24.63
15	6.77	24.48	16	6.95	24.30
17	6.92	24.13	18	7.31	23.96
19	6.79	21.67	20	7.02	23.33
21	7.33	25.00			

3.5 Discussion of results

The evaluation results for multiple reach channels show that the estimated values of "n" are fairly accurate for scenarios 7 and 8, where large number of measure-

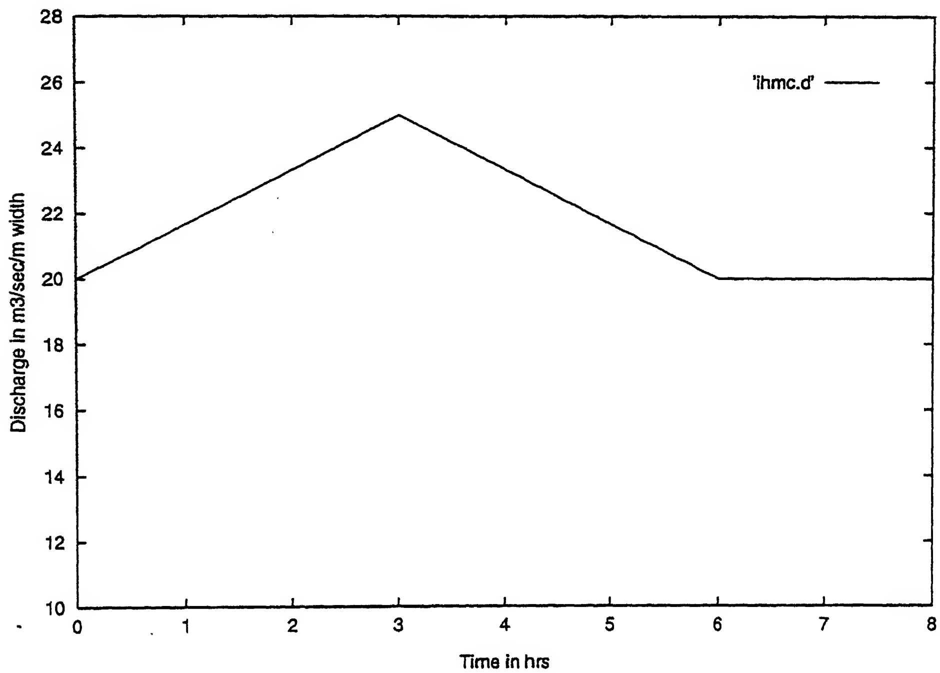

Figure 3.5: **Incoming flow hydrograph at the upstream end of the channel (for multiple reach channel)**

ment data points are used. For scenario 12, when only 16 data points are used, the estimation is not satisfactory. Also, bounds specified on "n" values, are artificially large enough to eliminate the influence on the solutions. These evaluation results indicate that accurate parameter estimation can be made, if the number of data representing measured flow variables are available for a number of spatial locations and time periods. This requirement is relevant especially when many parameters are to be estimated simultaneously. Since this model uses the embedding approach, it has the potential applicability to find other parameters such as bottom-slope, exponent of hydraulic radius etc., by treating them as variables in the optimization model. The initial estimate of "n" input to the optimization model was found to have insignificant effect on the solution results, although the global optimality cannot be guaranteed.

Table 3.5: **Scenarios Studied for Multiple reach Channel**

Scenario No.	No. of data points of depth (y)	No. of data points of discharge (q)	Estimated value of "n" at different reaches of channel	Actual value of "n" at different reaches of channel
7	21	21	0.02486 0.02245 0.01887 0.01808 0.01955 0.02195	0.024 0.025 0.021 0.018 0.019 0.023
8	17	17	0.02338 0.02368 0.0191 0.01808 0.0201 0.02185	0.024 0.025 0.021 0.018 0.019 0.023
9	10	10	0.02359 0.02301 0.01996 0.01893 0.02095 0.02159	0.024 0.025 0.021 0.018 0.019 0.023

Table 3.6: **Scenarios Studied for Multiple reach channel (contd.)**

Scenario No.	No. of data points of depth (y)	No. of data points of discharge (q)	Estimated value of "n" at different reaches of channel	Actual value of "n" at different reaches of channel
10	21	0	0.02376 0.02219 0.01866 0.01757 0.01857 0.02185	0.024 0.025 0.021 0.018 0.019 0.023
11	0	21	0.02190 0.02579 0.01711 0.02089 0.01671 0.02174	0.024 0.025 0.021 0.018 0.019 0.023
12	8	8	6.89E-06 3.76E-02 0.01828 0.01770 0.02005 0.02254	0.024 0.025 0.021 0.018 0.019 0.023

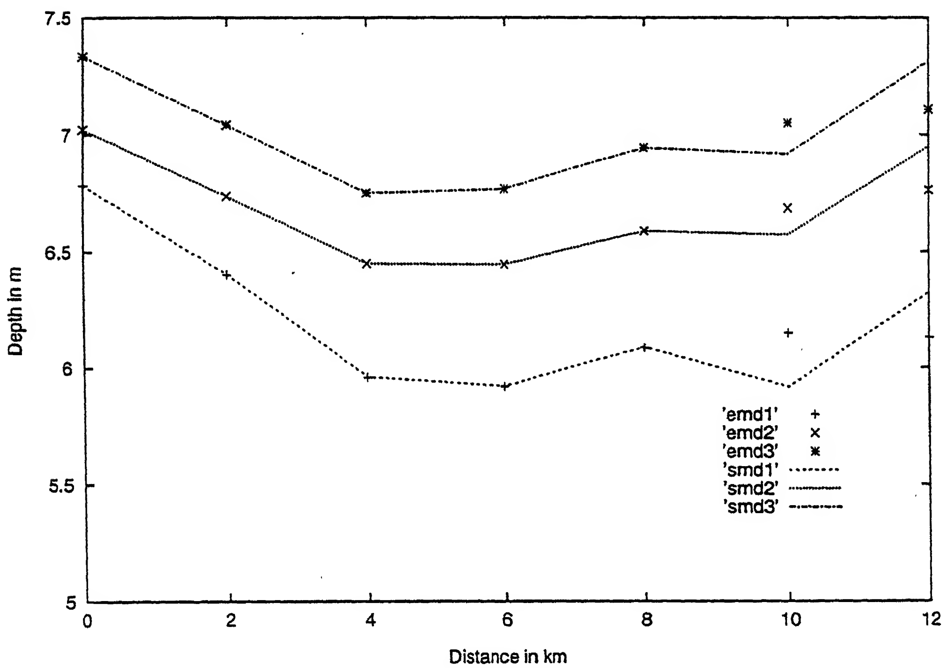

Figure 3.6: **Comparison of flow depth as observed (Simulated) and obtained as solution to the optimization model (Multiple reach, Scenario:7)**

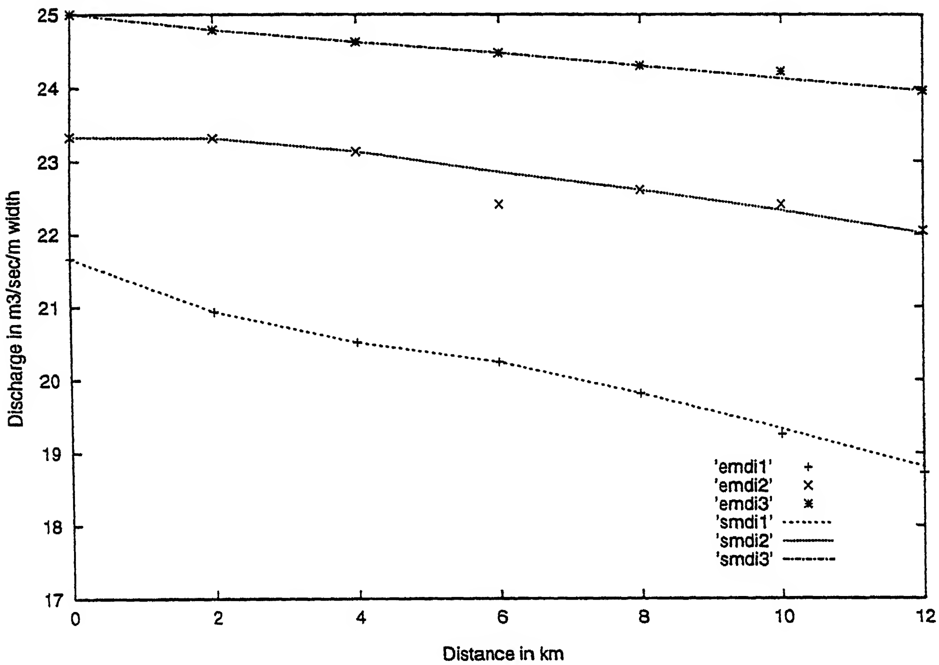

Figure 3.7: Comparison of discharge as observed (Simulated) and obtained as solution to the optimization model (Multiple reach, Scenario:7)

Table 3.7: Notation used in fig.(3.5) and fig.(3.6)

Notation used	Represents for a Multiple Reach Channel
emd1	Depth estimated for 1st time step
emd2	Depth estimated for 2nd time step
emd3	Depth estimated for 3rd time step
smd1	Simulated measured depth for 1st time step
smd2	Simulated measured depth for 2nd time step
smd3	Simulated measured depth for 3rd time step
emdi1	Discharge estimated for 1st time step
emdi2	Discharge estimated for 2st time step
emdi3	Discharge estimated for 3st time step
smdi1	Simulated measured discharge for 1st time step
smdi2	Simulated measured discharge for 2nd time step
smdi3	Simulated measured discharge for 3rd time step

Chapter 4

SUMMARY AND CONCLUSION

A mathematical model incorporating the open-channel unsteady flow equations within optimization model is developed for estimating the Mannings "n" in a single and multiple reach channel system. The discretization of the flow equations is carried out using the priessmann implicit finite-difference scheme. This model has the potential to estimate parameters like the bed-slope, exponent of hydraulic radius or any other parameter, by including the parameter to be estimated as the variable in the embedded optimization model. The performance of the developed model is evaluated for a number of scenarios. Some of the important conclusion are:

1. The potential applicability of the developed model is demonstrated using illustrative unsteady open channel flow problem.

2. As the number of the data points increases, the estimated value of the parameter "n" is very close to the actual value.

3. The embedded model is useful especially to estimate a number of parameters simultaneously, which otherwise may become a tedious task when using the simulation model outside the optimization model.

Bibliography

[1] Barnes, H.H. Jr.,"Roughness Characteristics of Natural Channels," U.S. Geological Survey Water-Supply paper 1849, Washington, 1967.

[2] Becker, L., and Yeh, W. W-G., "Identification of Parameters in Unsteady Open Channel Flows," *Water Resources Research,* Vol. 8, No.4, Aug., 1972, pp. 956-965.

[3] Becker, L., and Yeh, W. W-G., "The Identification of Multiple Reach Channel Parameters," *Water Resources Research,* Vol. 9, No. 2, Apr., 1973a, pp. 326-335.

[4] Becker, L., and Yeh, W. W-G., "Linear Programming and Channel Flow Identification," *Journal of the Hydraulics Division,* Vol. 99, No. HY11, Nov., 1973b, pp. 2013-2021.

[5] Chaudhry, M.H., "*Open Channel Flow,*" Prentice Hall of India Private Limited, New Delhi, 1993.

[6] Chow, V.T., "*Open channel Hydraulics,*" McGraw-Hill Book Company, New York, 1959.

[7] Dennis, J.E., Jr. and Schnabel, R.B., "*A New Derivation of Symmetric Positive Definite Secant Updates,*" In: 'Nonlinear Programming 4', O.L. Mangasarian, R.R. Meyer and S.M. Robinson, (eds), Academic Press, London and New York, pp. 167-199, 1981.

[8] Fread, D.L., and Smith, G.G., "Calibration techniques for 1-D unsteady flow models," *Journal of Hydraulic Division,* Vol. 104, No. 7, Jul., 1978, pp. 1027-1043.

[9] French, R.H., "*Open Channel Hydraulics,*" Chap. 4., Development of Uniform flow concepts, McGraw-Hill Book Company, New York, 1985, pp. 111-162.

[10] Garay, H.L., Haimes, Y.Y., and Das, P., "Distributed Parameter Identification of Ground Water Systems by Nonlinear Estimation," *Journal of Hydrology,* Vol. 30, No. 1/2, May, 1976, pp. 47-61.

[11] Kleinecke, D., "Use of Linear Programming for Estimating Geohydrologic Parameters of Groundwater Basins," *Water Resources Research,* Vol.7, No. 2, Apr., 1971, pp. 367-374.

[12] Neuman, S.P., "Calibration of Distributed Parameter Groundwater Flow Models Viewed as a Multiple-Objective Decision Process under Uncertainity," *Water Resources Research,* Vol. 9, No. 4, Aug., 1973, pp. 1006-1021.

[13] Numerical Algorithm Group NAG, E04UCF - NAG Fortran Library Routine Document, *Mark 14,* London, 1990.

[14] Powell, M.J.D., "*Introduction to Constrained Optimization,*" In: 'Numerical Methods for Constrained Optimization,' P.E. Gill and W. Murray, (eds). Academic Press, London and New York, pp. 1-28, 1974.

[15] Powell, M.J.D., "*Variable Metric Methods for Constrained Optimization.*" In: 'Mathematical Programming: The state of the art,' A. Bachem, M. Grötschel and B. Korte, (eds), Springer-Verlag, Berlin, Heidelberg, New york and Tokyo, pp. 288-311, 1983.

[16] Rao, S.S., "*Optimization, Theory and Application,*" Wiley-Eastern Limited. New Delhi, 1984.

[17] Subramanya, K., "*Flow in Open Channels,*" Tata McGraw-Hill Publishing Company Limited, New Delhi, 1984.

[18] Urquhart, W.J., "Hydraulics," Engineering Field Manual, *U.S. Department of Agriculture, Soil Conservation Service,* Washington, 1975.

[19] Wasantha Lal, A.M., "Calibration of Riverbed Roughness," *Journal of Hydraulic Engineering,* Vol. 121, No. 9, Sep., 1995, pp. 664-671.

[20] Yeh, W. W.-G., and Tauxe, G.W., "Optimal Identification of Aquifer Diffusivity Using Quazilinearization," *Water Resources Research,* Vol. 7, No. 4, Aug., 1971, pp. 955-962.